Received: *August 31, 1998*, Accepted: *September 14, 1998*

Supersymmetric contributions to rare kaon decays: beyond the single mass-insertion approximation.

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Abstract: We analyze the contributions to rare kaon decays mediated by flavor-changing Z-penguin diagrams in a generic low-energy supersymmetric extension of the Standard Model. In order to perform a model-independent analysis we expand the squark mass matrices around the diagonal, following the so called mass-insertion approximation. We argue that in the present case it is necessary to go up to the second order in this expansion to take into account all possible large effects. The current bounds on such second-order term, which was neglected in previous analyses, are discussed in detail and the corresponding upper bounds for the rare kaon decay rates are derived. As a result, we show that supersymmetric effects could lead to large enhancements of $K \to \pi \nu \bar{\nu}$ and $K_L \to \pi^0 e^+ e^-$ branching ratios.

Keywords: Supersymmetric Standard Model, Rare Decays, Kaon Physics.

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JHEP09(1998)009

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1. Introduction

Flavor-changing neutral-current (FCNC) processes provide a powerful tool for indirect searches of New Physics. This is particularly true in the framework of low energy supersymmetry [1], which represents one of the most interesting extensions of the Standard Model (SM). The large number of new particles carrying flavor quantum numbers, present in this context, would naturally lead to sizable effects in FCNC transitions [2, 3].

At the one-loop level, supersymmetric contributions to FCNC amplitudes can be classified into three groups, according to the virtual particles in the loop: i) Higgs/W-quarks, ii) gluino-squarks and iii) chargino/neutralino-squarks. The first group contains the SM contributions as a particular subgroup, whereas ii) and iii) represent genuine supersymmetric effects. Among them, gluino-squark transitions have been widely discussed in the literature [3, 4, 5, 6] and are expected to produce the dominant non-SM effect in $\Delta F = 2$ processes. This is confirmed by the analysis of $K^0 - \bar{K}^0$ mixing with the inclusion of gluino-squark contributions, which provides severe constraints on supersymmetric models [3, 4, 5, 6]. The effect of chargino/neutralino-squark diagrams is usually neglected in the analysis of such processes.

A different situation occurs in $\Delta F = 1$ transitions mediated by Z-penguin diagrams, which are particularly relevant to rare kaon decays, like $K \to \pi\nu\bar{\nu}$. As recently discussed in [7, 8], the dominant supersymmetric contribution to these processes is given by chargino-up-squarks diagrams. This is because the $Z\bar{q}_i q_j$ effective vertex is necessarily proportional to $SU(2)_L$-breaking couplings that, in supersymmetric models, are provided by $q_L - q_R$, $\tilde{q}_L - \tilde{q}_R$ and wino-higgsino mixing. Since the $\tilde{q}_L - \tilde{q}_R$ mixing in the down sector is suppressed by the small down-type Yukawa couplings, the effect of gluino and neutralino diagrams is necessarily small. On the other hand, the large Yukawa coupling of the top leads to potentially large effects in diagrams involving up-type quarks or squarks. Indeed the $(d,s)_L - t_R$ mixing, already present in the SM, is responsible for the m_t^2 enhancement of the Higgs/W-quark contribution to the $Z\bar{s}d$ effective vertex. Analogously, it is natural to expect a large effect due to the $(\tilde{d},\tilde{s})_L - \tilde{t}_R$ mixing in diagrams involving charginos and up-type squarks.

This effect has been already noted by Buras, Romanino and Silvestrini in the calculation of the supersymmetric contributions to $K \to \pi\nu\bar{\nu}$ [8]. However, this calculation has been performed in the *single* mass-insertion approximation, where only terms with at most one off-diagonal element of the squark mass matrix are considered. We believe that this approximation is not sufficient to fully account for possible large effects in the present case. Indeed, in order to provide the necessary $SU(2)_L$ breaking, at least two mass-mixing terms are necessary, either from the squark sector ($\tilde{u}_L - \tilde{u}_R$) or from the chargino sector (wino-higgsino). Since both these couplings vanish as $\mathcal{O}(m_W/M_S)$ in the limit of a heavy supersymmetry-breaking scale (M_S), we consider more appropriate to expand in both of them up to the second order. One could argue that wino-higgsino mixing is not suppressed by the off-diagonal flavor structure. However, the hierarchy of the Yukawa couplings implies that terms with a single wino-higgsino mixing always appear together with suppressed CKM factors. As a result, it is reasonable to expect that terms with a double LR mass insertion and without any CKM suppression are at least of the same order as those generated by a single LR mass insertion together with wino-higgsino mixing.

In the present paper we present a complete discussion of the supersymmetric contributions to the $Z\bar{s}d$ amplitude, beyond the single mass-insertion approximation. We find that the contribution generated by a double LR mass insertion in the up-squark sector, which was neglected in previous analyses, yields a potentially large effect. Employing the notation of [8], we can naively say that in this case the $\lambda_t m_t^2/m_W^2$ factor of the SM amplitude gets replaced by $(M_U^2)_{s_L t_R}(M_U^2)_{t_R d_L}/M_S^4$. Interestingly, this kind of mechanism is only weakly constrained by $K^0 - \bar{K}^0$ mixing and can provide a sizable enhancement (up to two orders of magnitude) to rare decay widths.

The paper is organized as follows. In section 2 we discuss the supersymmetric contributions to the $Z\bar{s}d$ amplitude, with particular attention to the hierarchy of the

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various terms. The role of box diagrams in $\Delta F = 1$ transitions is also briefly analyzed. In section 3 we discuss theoretical and phenomenological bounds on the up-type LR couplings. In section 4 we analyze the possible enhancements of rare kaon decays rates driven by these supersymmetric effects. The results are summarized in the conclusions.

2. The $Z\bar{s}d$ effective vertex

The amplitude we are interested in here is the one-loop FC effective coupling of the Z boson to down-type quarks, in the limit of vanishing external masses and momenta. As already emphasized in [7, 8], the $SU(2)_L \times U(1)_Y \to U(1)_{e.m.}$ gauge structure implies that this coupling proceeds through symmetry-breaking terms and involves only left-handed quarks. Thus it can be generally described by introducing the effective Lagrangian

$$\mathcal{L}_{FC}^{Z} = \frac{G_F}{\sqrt{2}} \frac{e}{2\pi^2} M_Z^2 \frac{\cos\Theta_W}{\sin\Theta_W} W_{ds}\, Z_\mu \bar{s}\gamma^\mu (1-\gamma_5) d \,+\, \text{h.c.}\,, \tag{2.1}$$

where W_{ds} is a complex dimensionless coupling.

In our conventions, the SM contribution of top-quark penguin diagrams, evaluated in the 't Hooft-Feynman gauge, leads to

$$W_{ds}^{SM} = \lambda_t C(x_{tW})\,, \tag{2.2}$$

where $\lambda_t = V_{ts}^* V_{td}$, V_{ij} are the CKM matrix elements [9] and $x_{tW} = m_t^2/m_W^2$. The loop function $C(x)$, originally computed in [10], can be found in the appendix. We recall that $C(x) \to x/8$ for large x.

In the minimal supersymmetric extension of the SM, which requires two Higgs doublets, the contribution of penguin diagrams with the exchange of charged Higgs and top-quark is aligned with the SM one (i.e. is proportional to λ_t). Denoting as usual by $\tan\beta$ the ratio of the two Higgs vacuum expectation values [1], we find

$$W_{ds}^{H} = \lambda_t \frac{m_H^2}{m_W^2 \tan^2\beta} H(x_{tH})\,, \tag{2.3}$$

where now $x_{tH} = m_t^2/m_{H^\pm}^2$. Similarly to the SM case, also $H(x) \to x/8$ for large x (the full expression of $H(x)$ is given in the appendix). The sum of (2.2) and (2.3) complete the first class of contributions outlined in the introduction, namely the Higgs/W-quark diagrams. To analyze the genuine supersymmetric effects, and particularly those generated by chargino-squark exchange, we first need to discuss shortly the structure of the supersymmetric mass matrices.

In the basis of the electroweak eigenstates, wino and higgsino, the chargino mass matrix is given by

$$M_\chi = \begin{pmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{pmatrix}, \tag{2.4}$$

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where the index 1 of both rows and columns refers to the wino state. Following the standard notation [1], here μ denotes the Higgs quadratic coupling and M_2 the soft supersymmetry-breaking wino mass. To define the mass eigenstates we introduce the unitary matrices $\hat{U}$ and $\hat{V}$ which diagonalize M_χ

$$\hat{U}^* M_\chi \hat{V}^\dagger = \mathrm{diag}(M_{\chi_1}, M_{\chi_2})\,. \tag{2.5}$$

As can be noticed, the off-diagonal entries of M_χ are $\mathcal{O}(m_W)$, whereas M_2 is $\mathcal{O}(M_S)$. In the limit where m_W/M_2 is a small parameter we can perform a perturbative diagonalization of M_χ around its diagonal elements, or, correspondingly, an expansion of $\hat{U}$ and $\hat{V}$ around the identity matrix.

In the squark sector we have 6×6 matrices which mix the three families of left-handed and right-handed squarks. A convenient basis for our calculation is the basis where the $d_L^i - \tilde{u}_L^j - \chi_n$ coupling is flavor diagonal and the $d_L^i - \tilde{u}_R^j - \chi_n$ one is ruled by the CKM matrix (see [8] for a more detailed description). In this case, the up-squark mass matrix is given by the Hermitian matrix

$$\mathcal{M}_U^2 = \begin{pmatrix} (M_U^2)_{d_L d_L} & (M_U^2)_{d_L u_R} \\ (M_U^2)_{u_R d_L} & (M_U^2)_{u_R u_R} \end{pmatrix}\,, \tag{2.6}$$

where the subscript d_L (which runs over three values) indicates the combination of left-handed up-type squarks which appear in the diagonal couplings $d_L - \tilde{u}_L^{(d)} - \chi_n$. On the other hand, the index u_R denotes the combination of right-handed up-type squarks which appear in the $d_L^i - \tilde{u}_R^j - \chi_n$ vertices ruled by the CKM matrix-element V_{ij}. Since $\mathcal{M}_U^2$ is Hermitian we need to introduce only one unitary matrix, $\hat{H}$, to diagonalize it

$$\hat{H}\mathcal{M}_U^2\hat{H}^\dagger = \mathrm{diag}\left(M^2_{\tilde{u}_1}, M^2_{\tilde{u}_2}, \ldots, M^2_{\tilde{u}_6}\right). \tag{2.7}$$

Also for $\mathcal{M}_U^2$ the off-diagonal elements are expected to be small and the perturbative diagonalization is well justified [3].

We are now ready to evaluate the contribution of the chargino-squark penguin diagrams in fig. 1. The full result before any mass expansion is quite simple and is given by[1]

$$W_{ds}^\chi = \frac{1}{8} A_{jl}^d \bar{A}_{ik}^s F_{jilk}\,, \tag{2.8}$$

where

$$A_{jl}^d = \hat{H}_{ld_L}\hat{V}_{1j}^\dagger - g_t V_{td}\hat{H}_{lt_R}\hat{V}_{2j}^\dagger\,, \tag{2.9}$$

$$\bar{A}_{ik}^s = \hat{H}_{s_L k}^\dagger \hat{V}_{i1} - g_t V_{ts}^* \hat{H}_{t_R k}^\dagger \hat{V}_{i2}\,, \tag{2.10}$$

$$\begin{aligned} F_{jilk} = {} & \hat{V}_{j1}\hat{V}_{1i}^\dagger\,\delta_{lk}\,k(x_{ik},x_{jk}) - 2\hat{U}_{i1}\hat{U}_{1j}^\dagger\,\delta_{lk}\,\sqrt{x_{ik}x_{jk}}\,j(x_{ik},x_{jk}) - \\ & -\delta_{ij}\,\hat{H}_{kq_L}\hat{H}_{q_L l}^\dagger\,k(x_{ik},x_{lk})\,. \end{aligned} \tag{2.11}$$

[1]The sum over the repeated indices i and j (running from 1 to 2), l and k (running from 1 to 6), and q_L (running over the three values d_L, s_L and b_L) is understood.

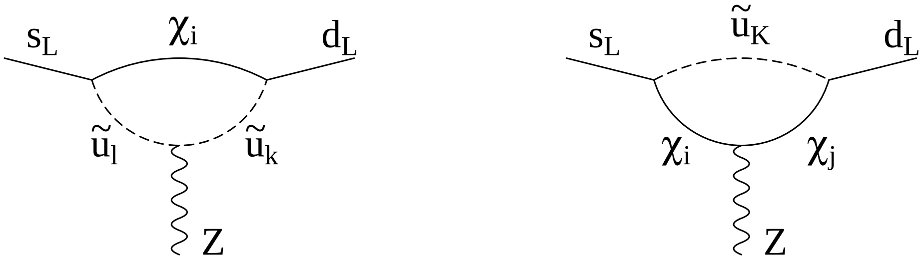

Figure 1: Chargino-up-squark penguin diagrams contributing to the $Z\bar{s}d$ effective vertex (diagrams involving self-energy corrections to the external legs are not explicitly shown).

For simplicity the effect of all the Yukawa couplings but $g_t = m_t/(\sqrt{2}m_W \sin\beta)$ has been neglected. Analogous to the previous cases, the variables x_{ij} denote ratios of squared masses (e.g. $x_{ik} = M^2_{\chi_i}/M^2_{\tilde{u}_k}$) and the functions $k(x,y)$ and $j(x,y)$ [8] can be found in the appendix.

The product of A^d_{jl} and $\bar{A}^s_{ik}$ in (2.8) generate four independent terms, proportional to $g_t^2\lambda_t$, $g_t V_{td}$, $g_t V^*_{ts}$ and 1, respectively, which correspond to the so-called RR, LR, RL and LL contributions in the notation of [7, 8]. We proceed to analyze these terms separately.

1. The RR contribution is generated by a Yukawa-type interaction in both quark-squark-chargino vertices of fig. 1. This term is the only one which survives in the limit of diagonal $\mathcal{M}^2_U$, i.e. to the lowest order in the perturbative expansion of $\hat{H}$ around unity. In this limit $W^{\chi}_{ds}|_{RR}$ is given by

$$W^{\chi}_{ds}|^0_{RR} = \frac{1}{8} g_t^2 \lambda_t \hat{V}^\dagger_{2j} \Big[\hat{V}_{j1} \hat{V}^\dagger_{1i} k(x_{it_R}, x_{jt_R}) - \\ -2\hat{U}_{i1}\hat{U}^\dagger_{1j} \sqrt{x_{it_R} x_{jt_R}}\, j(x_{it_R}, x_{jt_R}) \Big] \hat{V}_{i2}\,. \qquad (2.12)$$

As anticipated in the introduction, the $SU(2)_L$ breaking of the $Z\bar{s}d$ vertex requires at least two mass-mixing terms, either from the squark sector or from the chargino sector. In (2.12) the absence of the former mechanism implies a double wino-higgsino mixing, as can be easily checked by the mismatch of $\hat{V}$ and $\hat{U}$ indices. Thus $W^{\chi}_{ds}|^0_{RR}$ is parametrically suppressed by $\mathcal{O}(m_W^2/M_2^2)$ and aligned in phase with respect to W^{SM}_{ds}. To get a feeling of the numerical factors, note that $k(1,1) = 3/2$ and $j(1,1) = 1/2$. We then conclude that this contribution cannot provide a sizable effect, particularly in the limit of a heavy supersymmetry-breaking scale.

Considering higher orders in the perturbative expansion of $\hat{H}$, one can easily check that there is no contribution to $W^{\chi}_{ds}|_{RR}$ at the first order. At the second order it is possible to generate a non-vanishing contribution and also to avoid the wino-higgsino mixing. However, the unavoidable factor λ_t makes $W^{\chi}_{ds}|_{RR}$ always not particularly interesting with respect to W^{SM}_{ds}.

2. The LR and RL terms are originated by a Yukawa-type interaction in one of the two quark-squark-chargino vertices of fig. 1 and a gauge-type interaction in the other. As can be easily understood, this implies that $W^{\chi}_{ds}|_{LR}$ and $W^{\chi}_{ds}|_{RL}$ are at least of first order in both wino-higgsino and $\tilde{q}_L - \tilde{q}_R$ mixing. Performing explicitly the expansion of $\hat{H}$ up to the first order, as discussed in the appendix, we find

$$W^{\chi}_{ds}|^1_{LR} = -\frac{1}{8} g_t V_{td} \frac{(M^2_U)_{s_L t_R}}{M^2_{\tilde{u}_L}} \hat{V}^{\dagger}_{2j} \Big[\hat{V}_{j1} \hat{V}^{\dagger}_{1i} k(x_{iu_L}, x_{ju_L}, x_{t_R u_L}) - \\ -\delta_{ij} k(x_{iu_L}, x_{t_R u_L}, 1) - 2\hat{U}_{i1} \hat{U}^{\dagger}_{1j} \sqrt{x_{iu_L} x_{ju_L}} j(x_{iu_L}, x_{ju_L}, x_{t_R u_L}) \Big] \hat{V}_{i1} \,, \qquad (2.13)$$

where $M_{\tilde{u}_L}$ indicates the average mass of the approximate left-handed up squarks. The $W^{\chi}_{ds}|^1_{RL}$ term can be obtained from (2.13) with the substitution $V_{td}(M^2_U)_{s_L t_R} \to V^*_{ts}(M^2_U)_{t_R d_L}$ and with the exchange $1 \leftrightarrow 2$ in the indices of the $\hat{V}$ matrices outside the square brackets.

The presence of a single CKM matrix element in (2.13) leads to potentially large effects: the missing factor V^*_{ts} is replaced by $(\delta^U_{LR})^*_{ts}$, where

$$(\delta^U_{LR})_{ab} = (M^2_U)_{a_R b_L} / M^2_{\tilde{u}_L} \,, \qquad (2.14)$$

and the ratio $(\delta^U_{LR})_{ts}/V_{ts}$ can be larger than one [5]. However, this enhancement is partially compensated by the $\mathcal{O}(m_W/M_2)$ suppression induced by the wino-higgsino mixing and the total effect is not very large. Indeed the phenomenological bounds on $(\delta^U_{LR})_{ts}$, dictated mainly by $b \to s\gamma$, become weaker for large supersymmetric masses, when the wino-higgsino suppression gets stronger. As a result, $W^{\chi}_{ds}|^1_{LR}$ can be at most as large as W^{SM}_{ds} [8]. Similar comments apply also to $W^{\chi}_{ds}|^1_{RL}$ (see the next section for a more detailed discussion about limits on $(\delta^U_{LR})_{ts}$ and $(\delta^U_{LR})_{td}$).

It is interesting to note how the LR and RL terms, which arise only at first order in the expansion of $\hat{H}$, are potentially larger and not aligned in phase with respect to the lowest order contribution, provided by $W^{\chi}_{ds}|^0_{RR}$. This is clearly a consequence of the disappearance of one of the two CKM factors. For this reason, it is natural to expect that terms arising at the second order in the mass-insertion expansion and without any CKM suppression could be even bigger.

3. The LL term is originated by a double gauge-type interaction in the quark-squark-chargino vertices of fig. 1. Similarly to the RR case, this implies a second-order mixing either in the chargino sector or in the $\tilde{q}_L - \tilde{q}_R$ sector. However, contrary to the RR case, there is no contribution to the leading-order in the expansion of $\hat{H}$. The first term in this expansion arises to the first order and is given by

$$W^{\chi}_{ds}|^1_{LL} = -\frac{1}{8} \frac{(M^2_U)_{s_L d_L}}{M^2_{\tilde{u}_L}} \hat{V}^{\dagger}_{1j} \Big[\hat{V}_{j2} \hat{V}^{\dagger}_{2i} k(x_{iu_L}, x_{ju_L}, 1) - \\ -2\hat{U}_{i2} \hat{U}^{\dagger}_{2j} \sqrt{x_{iu_L} x_{ju_L}} j(x_{iu_L}, x_{ju_L}, 1) \Big] \hat{V}_{i1} \,. \qquad (2.15)$$

As can be noted, this term involves a double wino-higgsino mixing, which provides the necessary $SU(2)_L$ breaking, and a first-order mixing among left-handed squarks. Thus, even if apparently enhanced by the absence of any CKM factor, $W^{\chi}_{ds}|^1_{LL}$ is strongly suppressed in the limit of heavy supersymmetry-breaking scale. Moreover, the $SU(2)_L$ invariance of the soft-breaking terms relates $(M^2_U)_{s_L d_L}$ to $(M^2_D)_{s_L d_L}$ [5], which is strongly constrained by $K^0 - \bar{K}^0$ mixing. As a result, $W^{\chi}_{ds}|^1_{LL}$ turns out to be always smaller than W^{SM}_{ds} [8].

A different scenario occurs if we consider the contribution to $W^{\chi}_{ds}|_{LL}$ which survives in the absence of wino-higgsino mixing. In this case one has to go at least to the second order in the expansion of $\hat{H}$, and only terms with a double LR mixing survive. The lowest-order result in this limit is simply given by

$$\begin{aligned} W^{\chi}_{ds}|^2_{LL} &= \frac{1}{8}\,\frac{(M^2_U)_{s_L q_R}(M^2_U)_{q_R d_L}}{m^4_{\chi_1}}\, l(x_{u_L 1}, x_{u_L 1}, x_{q_R 1}) \\ &= \frac{1}{8}\,(\delta^U_{LR})^*_{qs}\,(\delta^U_{LR})_{qd}\,x^2_{u_L 1}\, l(x_{u_L 1}, x_{u_L 1}, x_{q_R 1})\,, \end{aligned} \tag{2.16}$$

where the function $l(x,y,z)$, normalized to $l(1,1,1) = -1/12$, is reported in the appendix. Contrary to the cases of $W^{\chi}_{ds}|^0_{RR}$, $W^{\chi}_{ds}|^1_{LR}$ and $W^{\chi}_{ds}|^1_{LL}$ discussed previously, there is no explicit suppression in $W^{\chi}_{ds}|^2_{LL}$ in the limit of heavy superpartners. Actually the upper bounds on the factor $(\delta^U_{LR})^*_{qs}(\delta^U_{LR})_{qd}$ go to zero in this limit, as we will see in the next section. However, for $q = t$ there is room enough to produce sizable effects (in close analogy to the λ_t factor in the SM case) even for $M_S \sim 1$ TeV. In this case $W^{\chi}_{ds}|^2_{LL}$ could be substantially larger than W^{SM}_{ds} providing sizable enhancements to rare kaon decay rates.

Using the effective Lagrangian (2.1) we can easily calculate the effects of the $Z\bar{s}d$ penguins discussed above in various processes. In the case of $K \to \pi\nu\bar{\nu}$ decays we find that the contribution generated by (2.1) to the X function, defined by

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\,\frac{\alpha}{2\pi\sin^2\Theta_W}\,\lambda_t X\,\bar{s}\gamma^\mu(1-\gamma_5)d\,\bar{\nu}_l\gamma^\mu(1-\gamma_5)\nu_l\, + \text{h.c.}, \tag{2.17}$$

is given by

$$X_{Z\bar{s}d} = W_{ds}/\lambda_t\,. \tag{2.18}$$

Comparing our results in (2.3), (2.12), (2.13) and (2.15) with those reported in the appendix of [8] we find a perfect agreement but for an overall factor $-1/2$ which is a mere misprint.[2]

In principle, in the case of $K \to \pi\nu\bar{\nu}$ decays also supersymmetric box diagrams could provide sizable effects, as it happens for instance in the SM case [11]. However, the contribution of chargino-up-squark box diagrams to X turns out to be always

[2] This misprint does not affect the numerical results of [8]. We thank L. Silvestrini for clarifying this point.

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suppressed by a factor $m_W^2/M_{\tilde{q}}^2$, besides possible wino-higgsino mixing.[3] In a generic expansion in powers of off-diagonal mass terms, denoted by ϵ, the box contribution to X starts at $\mathcal{O}(\epsilon^3)$, whereas the penguin one at $\mathcal{O}(\epsilon^2)$. Thus, in general we agree with the statement of Nir and Worah [7] that penguin contributions provide the dominant effect. Only in the case of the terms proportional to $(M_U^2)_{s_L d_L}$, when the penguin contribution is suppressed and starts at $\mathcal{O}(\epsilon^3)$, the corresponding box term turns out to be competing [8]. However, as long as we are interested only in possible large effects this is not a relevant case.

Similar arguments apply also to other processes where the effective $Z\bar{s}d$ vertex can contribute, like $K \to \ell^+\ell^-$ and $K \to \pi\ell^+\ell^-$ decays. Hence, in a minimal supersymmetric extension of the SM with generic flavor sector, and particularly in the limit of a heavy supersymmetry-breaking scale, we consider it a good approximation to encode the dominant non-SM effects to these processes via the Lagrangian (2.1). A similar approach was considered by Nir and Silverman in a different context [12]: the coupling W_{ds} in (2.1) is related to the U_{ds} of Nir and Silverman by

$$U_{ds} = \frac{\alpha}{\pi \sin^2 \Theta_W} W_{ds} \,. \tag{2.19}$$

3. Bounds on the $(\delta_{LR}^U)_{ij}$ couplings

In the previous section we have argued that the (LR)^2 term that appears at second order in the mass-insertion expansion, may give the largest enhancement to the $Z\bar{s}d$ effective vertex with respect to the SM contribution. In the present section we will analyze in detail the bounds we can put on this term, considering both phenomenological information, and purely theoretical constraints.

3.1. Vacuum-stability bounds

Before analyzing the bounds coming from phenomenology, we discuss an interesting result obtained by Casas and Dimopoulos [13], who have shown that bounds on the off-diagonal LR entries of the squark mass matrices can be derived also from the requirement that the standard vacuum of the theory be stable. In particular they require that there are no charge and color breaking minima (CCB bounds), nor directions in which the potential is unbounded from below (UFB bounds). Obviously, these bounds have to be satisfied by any model (and interestingly enough, are generally *not* satisfied). The only way to avoid or at least to soften these constraints is to assume that we live in a sufficiently long-lived metastable vacuum. However, to be more conservative, we will not take into account this possibility. The consequence of the stability bounds for

[3]The contribution of the charged-Higgs box diagrams is clearly negligible because of the small lepton Yukawa couplings.

the matrix elements of our interest can be stated in a very simple manner:

$$\left|(\delta^U_{LR})_{ij}\right| \le m_{u_k} \frac{\sqrt{2M^2_{\tilde u} + M^2_{\tilde l}}}{M^2_{\tilde u}} \qquad (k = \max(i,j))\,, \tag{3.1}$$

where $M^2_{\tilde u}$ and $M^2_{\tilde l}$ denote the average masses of up-squarks and sleptons, respectively, and m_{u_k} indicates the mass of the u_k quark. The m_{u_k} factor provides a very strong suppression unless one of the two generation indices (i and j) is equal to 3. For this reason it is a good approximation to replace the sum $\sum_q (\delta^U_{LR})^*_{qs}\,(\delta^U_{LR})_{qd}$ in (2.16) with the product $(\delta^U_{LR})^*_{ts}\,(\delta^U_{LR})_{td}$. In this case the bound (3.1) can be roughly expressed in the following form

$$\left|(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right| \le \frac{3m_t^2}{M_S^2}\,, \tag{3.2}$$

where with M_S we have indicated a typical supersymmetric scale. Actually the bound (3.1) corresponds to the UFB constraint, but as long as we consider almost degenerate supersymmetric particles CCB and UFB bounds are essentially equivalent [13].

At this point it is useful to make a first estimate of the possible enhancement induced by the (LR)^2 mass insertion in the $Z\bar{s}d$ vertex. Comparing (2.2) and (2.16), in the limit $x_{tW} \gg 1$ and assuming almost degenerate supersymmetric particles, leads to

$$\left|\frac{W^\chi_{ds}|^2_{LL}}{W^{SM}_{ds}}\right| \simeq \left|\frac{(\delta^U_{LR})^*_{ts}\,(\delta^U_{LR})_{td}}{12\,x_{tW}\,\lambda_t}\right| \lesssim 20 \times \left(\frac{500\mathrm{GeV}}{M_S}\right)^2\,, \tag{3.3}$$

where the last inequality has been obtained imposing the bound (3.2). As can be noticed, though stringent the model-independent constraint leaves enough room for a large enhancement, even for M_S as large as 1 TeV.

3.2. Box $\Delta S = 2$

A term with two LR mass insertions appears in the box diagram (containing charginos and squarks) contributing to $K^0 - \bar{K}^0$ mixing. In this case, however, this term appears only at a subleading level. The complete expression for the contribution of the box diagram to the effective Hamiltonian for $\Delta S = 2$ is

$$\mathcal{H}^{\rm eff}_{\Delta S=2} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi\sin^2\Theta_w}\,A^d_{ik}\bar{A}^s_{jk}A^d_{jl}\bar{A}^s_{il}\,\frac{m_W^2}{M^2_{\tilde q_k}}\,k(x_{ik},x_{jk},x_{lk})(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L)\,. \tag{3.4}$$

We may now expand the mass matrices around their diagonal part to the desired order. Considering only terms without chargino mixing, we get

$$\begin{aligned} &A^d_{ik}\bar{A}^s_{jk}A^d_{jl}\bar{A}^s_{il}\,k(x_{ik},x_{jk},x_{lk}) = \\ &\quad -\frac{1}{20}\left[((\delta^U_{LL})_{sd})^2 - \frac{2}{3}(\delta^U_{LL})_{sd}((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}) + \frac{1}{7}((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td})^2 + \ldots\right] - \\ &\quad -\frac{g_t^2\lambda_t}{10}\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td} + \ldots\right) + \mathcal{O}(\lambda_t^2)\,. \end{aligned} \tag{3.5}$$

To obtain this result we have not only applied the formulae for the perturbative diagonalization of the mass matrices (that we give in appendix), but have also taken the limit where all superpartners have approximately the same mass ($x_{ki} = 1$ for all k's and i's). If we now use the experimental information on $\Delta m_K = 3.5 \times 10^{-12}$, and require that the contribution of the term with two LR insertions in (3.5)[4] does not exceed the experimental value, we get

$$\sqrt{\mathrm{Re}\left[\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right)^2\right]} \leq 0.16 \times \left(\frac{M_S}{500\mathrm{GeV}}\right) \quad . \tag{3.6}$$

We remark that this limit is derived using the quadratic term in (3.5), as the linear one is multiplied by λ_t which suppresses its contribution strongly. Similarly, we have not considered the bounds that could be obtained on the single $(\delta^U_{LR})_{ts}$ and $(\delta^U_{LR})_{td}$ couplings, which always appear suppressed both by CKM factors and chargino mixing. Of course this limit is rather generous, as one would expect the first two terms in (3.5) to be responsible for the main part of the effect. On the other hand, until we will be able to get some independent information on the first two terms in the expansion (and on their signs too) this is the best we can get from this quantity.

If we now look at the imaginary part of the same matrix element, and consider the experimental information on $\mathrm{Re}(\epsilon)$, we can get a bound on the imaginary part of the $(\mathrm{LR})^2$ term squared:

$$\sqrt{\mathrm{Im}\left[\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right)^2\right]} \leq 0.015 \times \left(\frac{M_S}{500\mathrm{GeV}}\right) \quad . \tag{3.7}$$

3.3. Limits from B and D physics

Buras, Romanino and Silvestrini [8] have analyzed the bounds on various mass insertions coming from B-meson phenomenology. From the chargino contribution to $B_d - \bar{B}_d$ mixing they get

$$\left|(\delta^U_{LR})_{dt}\right| \leq 0.1 \times \left(\frac{M_S}{500\mathrm{GeV}}\right) . \tag{3.8}$$

A bound on the other matrix element of our interest was earlier obtained by Misiak, Pokorski and Rosiek analyzing the chargino contribution to $b \to s\gamma$ [5]:

$$\left|(\delta^U_{LR})_{st}\right| \leq 3 \times \left(\frac{M_S}{500\mathrm{GeV}}\right)^2 . \tag{3.9}$$

In principle a limit on $\mathrm{Re}[((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td})^2]$, similar to the one in (3.6), could be obtained from the analysis of the gluino-up-squark box diagram contributing to $D^0 - \bar{D}^0$ mixing. Note, however, that this bound is very different from those discussed above since it can be made arbitrarily small in the limit of a heavy gluino mass. Assuming gluino and chargino approximately degenerate, the constraint obtained by $D^0 - \bar{D}^0$

[4]Evaluating this with the approximation $\langle K^0|(\bar{s}_L\gamma^\mu d_L)(\bar{s}_L\gamma_\mu d_L)|K^0\rangle = 1/3\, m_K\, f_K^2$.

mixing is essentially equivalent to the $K^0 - \bar{K}^0$ one. Indeed the $(g_{\rm strong}/g)^4$ enhancement of the gluino box diagram with respect to the chargino one is almost completely compensated by the less stringent experimental constraint on Δm_D with respect to Δm_K.[5]

3.4. Bounds on the $Z\bar{s}d$ vertex

As anticipated in the previous section, the $Z\bar{s}d$ effective vertex contributes to various rare kaon transitions. Some of them have been observed, whereas stringent experimental limits exist on the others: we can therefore use these informations to derive bounds on the (LR)^2 term which we are now analyzing. These bounds are best expressed in terms of the coupling W_{ds} introduced in (2.1). A similar analysis has been already made by Grossman and Nir [14], using exactly the same language of an effective $Z\bar{s}d$ coupling (but using the U_{ds} of [12]). Following and partially updating (and correcting) their results, we find

1. from the process $K_L \to \mu^+\mu^-$ [15]:
$$|\mathrm{Re}(W_{ds})| \leq 2.2 \times 10^{-3}\,; \tag{3.10}$$
2. from[6] $B(K^+ \to \pi^+\nu\bar{\nu}) < 2 \times 10^{-9}$ [16]:
$$|W_{ds}| \leq 3.6 \times 10^{-3}\,; \tag{3.11}$$
3. from the measurement of ϵ:
$$|\mathrm{Re}(W_{ds})\mathrm{Im}(W_{ds})| \leq 1.1 \times 10^{-5}\,. \tag{3.12}$$

In principle similar bounds could be obtained from ϵ'/ϵ and $B(K_L \to \pi^0 e^+e^-)$. However, in both cases the large theoretical uncertainties and the poor experimental information lead to weaker constraints. To translate the results (3.10-3.12) into bounds for the (LR)^2 term of our interest we use the relation

$$W^{\chi}_{ds}|^2_{LL} = \frac{1}{96}(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\,,$$

derived from (2.16) in the limit of degenerate supersymmetric particles. We then obtain

$$\begin{aligned}
\left|\mathrm{Re}\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right)\right| &\leq 0.21\,,\\
\left|(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right| &\leq 0.35\,,\\
\left|\mathrm{Re}((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td})\mathrm{Im}((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td})\right| &\leq 0.1\,.
\end{aligned} \tag{3.13}$$

[5] We are grateful to M. Worah for a clarifying discussion about this point.
[6] Note that the corresponding bound in [14] was larger due to missing factor six in their Eq. (13).

3.5. Summary of the bounds

The two limits derived from the analysis of the $\Delta S = 2$ box diagram can be written as follows:

$$\left|\left[\mathrm{Re}\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right)\right]^2 - \left[\mathrm{Im}\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right)\right]^2\right| \leq 2.6\times 10^{-2}\times\left(\frac{M_S}{500\mathrm{GeV}}\right)^2 , \quad (3.14)$$

$$\left|\mathrm{Re}((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td})\mathrm{Im}((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td})\right| \leq 1.1\times 10^{-4}\times\left(\frac{M_S}{500\mathrm{GeV}}\right)^2 ; \quad (3.15)$$

those obtained from B physics lead to

$$\left|(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right| \leq 0.3\times\left(\frac{M_S}{500\mathrm{GeV}}\right)^3 , \quad (3.16)$$

whereas the model-independent one is given by

$$\left|(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right| \leq 0.3\times\left(\frac{500\mathrm{GeV}}{M_S}\right)^2 . \quad (3.17)$$

Finally, the 'scale-independent' limits derived from the phenomenological analysis of the $Z\bar{s}d$ vertex are

$$\begin{aligned}\left|\mathrm{Re}\left((\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right)\right| &\leq 0.21 ,\\ \left|(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right| &\leq 0.35 ,\end{aligned} \quad (3.18)$$

where we have skipped the bound on the product of real and imaginary part, which is clearly negligible with respect to the one in (3.15).

A summary of the various bounds is displayed in fig. 2, for the sample value $M_S =$ 500 GeV. From the figure it is clear that the bound in (3.15) is by far the most stringent one. This implies that, if we assume that real and imaginary parts of $(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}$ are of the same order, these are $\mathcal{O}(10^{-2})$. On the contrary, if one of the two is zero the other can be $\mathcal{O}(10^{-1})$. The maximum value allowed for the real part setting the imaginary part to zero, or vice versa, is 0.16 as dictated by the bound (3.14). Playing around with the M_S dependence of these bounds one can find that the maximum value allowed for either the imaginary or real part (when the other is set to zero) can grow up to 0.2 for $M_S = 600$ GeV, again a bound dictated by (3.14). Above this value of M_S the model-independent limit on the modulus becomes more stringent. Notice in fact that the model-independent limit on $\left|(\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td}\right|$ and the one coming from B-physics have opposite dependence on the average mass of the superpartners. So that for $M_S < 500$ GeV it is the B-physics one which dominates, whereas above 500 GeV the model-independent one takes over.

In conclusion, for $M_S \gtrsim 600$ GeV we can just consider the two bounds in (3.15) and (3.17), as all the others will be automatically satisfied. If we define

$$\tilde{\lambda}_t = |\tilde{\lambda}_t| e^{i\tilde{\theta}_t} = (\delta^U_{LR})^*_{ts}(\delta^U_{LR})_{td} , \quad (3.19)$$

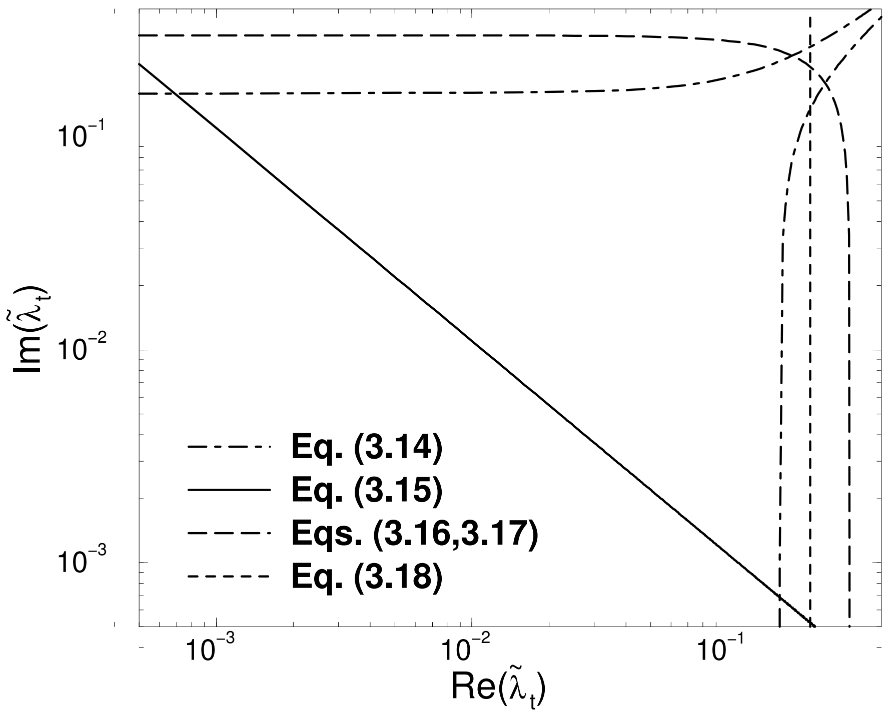

Figure 2: Summary of the bounds on the $(\mathrm{LR})^2$ coupling $\tilde{\lambda}_t$, as defined in eq. (3.19).

the bound we have to satisfy for $M_S \gtrsim 600$ GeV is

$$|\tilde{\lambda}_t| < \min\left[0.2 \times \left(\frac{600\mathrm{GeV}}{M_S}\right)^2, \frac{2\times 10^{-2}}{\sqrt{|\sin 2\tilde{\theta}_t|}}\left(\frac{M_S}{600\mathrm{GeV}}\right)\right], \tag{3.20}$$

whereas the phase $\tilde{\theta}_t$ is unbounded.

At this point one could argue whether a reasonable low-energy supersymmetric model could saturate this bound. It is beyond the scope of this paper to analyze any model in detail. However, we recall that in generic superstring scenarios the so-called A terms, responsible for the LR entries of the mass matrices, are expected to be $\mathcal{O}(M_S)$ [17]. This would imply $(M^2_{LR})_{ij} \sim \mathcal{O}(m_W \cdot M_S)$. Thus in general it is not unnatural to consider models where the bound (3.20) is saturated (see e.g. the discussion at the end of ref. [13]).

4. Phenomenological consequences of a large $Z\bar{s}d$ effective coupling

The most clear signature of an enhancement in the $Z\bar{s}d$ effective vertex could be found in $K \to \pi\nu\bar{\nu}$ decays. Within the SM these transitions can be described by means of the Hamiltonian (2.17), with the X function given by [11]

$$X_{SM} = X_t\left(x_{tW}\right) + \frac{\lambda^4\lambda_c}{\lambda_t}P_c\,, \tag{4.1}$$

where $X_t(x_{tW}) \simeq 1.5$ is generated by the dominant top-quark contribution (summing penguin and box diagrams) and $P_c = 0.40 \pm 0.06$ is due to the charm loops (as usual λ denotes the Cabibbo angle and $\lambda_c = V_{cs}^* V_{cd}$, thus $|\lambda^4 \lambda_c/\lambda_t| \sim \mathcal{O}(1)$). The branching ratios of K^+ and K_L modes can be expressed in terms of the X function as

$$BR(K^+ \to \pi^+ \nu\bar{\nu}) = \kappa_+ \left| \frac{\lambda_t X}{\lambda^5} \right|^2 , \tag{4.2}$$

$$BR(K_L \to \pi^0 \nu\bar{\nu}) = \kappa_L \left(\mathrm{Im} \frac{\lambda_t X}{\lambda^5} \right)^2 , \tag{4.3}$$

where $\kappa_+ = 4.11 \times 10^{-11}$ and $\kappa_L = 1.80 \times 10^{-10}$ [18]. For a numerical estimate we recall that $\lambda = 0.22$, $|\lambda_t| \simeq 3 \times 10^{-4}$ and $\mathrm{Im}\lambda_t \simeq |\lambda_t|/3$ [19].

In extensions of the SM where the main new-physics effects can be encoded via the effective coupling W_{ds}, we should add to X_{SM} the $X_{Z\bar{s}d}$ function defined in (2.18). Thus if we add to the SM contribution the dominant supersymmetric effect, provided by the $(\mathrm{LR})^2$ terms, we find

$$\begin{aligned} X_{tot} &= \frac{1}{8} \frac{\tilde{\lambda}_t}{\lambda_t} x_{u_L 1}^2 \, l(x_{u_L 1}, x_{u_L 1}, x_{q_R 1}) + X_t\left(x_{tW}\right) + \frac{\lambda^4 \lambda_c}{\lambda_t} P_c \\ &\simeq \frac{1}{96} \frac{\tilde{\lambda}_t}{\lambda_t} + X_t\left(x_{tW}\right) + \frac{\lambda^4 \lambda_c}{\lambda_t} P_c \, , \end{aligned} \tag{4.4}$$

where $\tilde{\lambda}_t$ has been defined in (3.19) and the second line of (4.4) is obtained in the limit of almost degenerate superpartners. Given the constraints on $|\tilde{\lambda}_t|$ reported in (3.20) it is clear that large enhancements with respect to the SM case are possible. In the rate of the charged mode one can gain up to an order of magnitude if M_S is around 600 GeV, where the effect is maximum. In the K_L case a crucial role is played by the new phase $\tilde{\theta}_t$: if $\tilde{\theta}_t \sim 90^\circ$ a huge enhancement (up to two order of magnitudes) is possible for a wide range of M_S. In table 1 we summarize the upper bounds on the two modes for $M_S \sim 0.6$ TeV and $M_S \sim 1$ TeV.

Related modes which could allow to detect an enhancement in the $Z\bar{s}d$ effective amplitude are the $K \to \pi \ell^+ \ell^-$ decays. In the charged channel the long-distance process $K^+ \to \pi^+ \gamma^* \to \pi^+ \ell^+ \ell^-$ is by far dominant, hiding the contribution of the $Z\bar{s}d$ transition. However, the single-photon exchange amplitude is forbidden by CP invariance in the $K_L \to \pi^0 \ell^+ \ell^-$ mode, which is therefore more sensitive to short-distance dynamics (see e.g. ref. [20] for a recent discussion about these decays). Assuming that both $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0 \nu\bar{\nu}$ transitions are dominated by the CP-violating part of the $Z\bar{s}d$ effective amplitude, we can easily relate their widths. Indeed, neglecting the electron mass and the effects of the small vector coupling of the electrons to the Z, leads to $\Gamma(K_L \to \pi^0 e^+ e^-) = \Gamma(K_L \to \pi^0 \nu\bar{\nu})/6$. Using this approximate relation we have derived the upper bound for $B(K_L \to \pi^0 e^+ e^-)$ reported in the last line of table 1. As it is well known, in addition to the direct CP-violating transition, the $K_L \to \pi^0 e^+ e^-$

decay mode	maximum SUSY branching ratio $M_S \sim 0.6$ TeV	$M_S \sim 1$ TeV	SM branching ratio
$K^+ \to \pi^+ \nu\bar{\nu}$	1×10^{-9}	4×10^{-10}	$(9.1 \pm 3.8) \times 10^{-11}$ [19]
$K_L \to \pi^0 \nu\bar{\nu}$	4×10^{-9}	6×10^{-10}	$(2.8 \pm 1.7) \times 10^{-11}$ [19]
$K_L \to \pi^0 e^+ e^-$	6×10^{-10}	1×10^{-10}	$\lesssim$ few $\times 10^{-11}$ [19, 20]

Table 1: Approximate upper bounds for the branching ratios of $K \to \pi\nu\bar{\nu}$ and $K_L \to \pi^0 e^+ e^-$ decays within the low-energy supersymmetric scenario discussed in the text, compared to the SM expectations.

decay can proceed through indirect CP-violation ($K_L \to K_S \to \pi^0 e^+ e^-$) or via the CP-conserving two-photon exchange ($K_L \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^-$). However, both these mechanisms are expected to produce corrections to $B(K_L \to \pi^0 e^+ e^-)$ at the level of few$\times 10^{-11}$ at most [19, 20]. This ensures that a detection of $B(K_L \to \pi^0 e^+ e^-)$ above 10^{-10} can be considered as a clear signature of new-physics.

As can be expected, the upper limits for the supersymmetric branching ratios shown in table 1 are much larger than those reported in [21], where the supersymmetric contributions to $K \to \pi\nu\bar{\nu}$ have been evaluated essentially without allowing $(\tilde{d}, \tilde{s})_L - \tilde{t}_R$ mixing. We stress, however, that our upper bounds are significantly larger also than those recently obtained in [8], where the $(\tilde{d}, \tilde{s})_L - \tilde{t}_R$ mixing has been evaluated only to first order within the mass-insertion approximation.

To conclude this section we emphasize that in these numerical results we did not take into account other possible sources of enhancement. Indeed we could have obtained even bigger effects playing with the various supersymmetric mass ratios (that we have set to 1 just to simplify our results). In particular, larger effects are obtained with a wino mass lighter than the average squark mass. Moreover, we have neglected possible constructive interferences between the leading $(\mathrm{LR})^2$ terms and the subleading, but still not negligible, LR terms. Finally, we have neglected possible destructive interferences between chargino- and gluino-mediated amplitudes when evaluating the bounds on the LR couplings induced by $K^0 - \bar{K}^0$ mixing: this effect could easily lead to overcome the stringent constraint in eq. (3.15).

5. Conclusions

In this paper we have analyzed supersymmetric contributions to rare K decays mediated by an effective $Z\bar{s}d$ vertex. We have adopted the strategy of the so-called mass-insertion approximation, which consists in assuming that the squark mass matrices are almost diagonal, and that their diagonalization can be performed perturbatively. While recent similar analyses have stopped this approximation to the first order, we have argued that in the present case it is necessary to go up to the second order in this expansion to account for all possible important effects.

This result does not contradict the validity of the mass-insertion approximation. Rather, we have stressed the fact that there is an interplay between the squark mass matrices and other mass matrices present in the theory. The reason why the second-order terms in this expansion can be more important than the first-order ones, is because they do not contain anymore off-diagonal CKM matrix elements which are known to be suppressed. In other words, we could say that all mass matrices (both those of the quarks and of the squarks) in the supersymmetric theory are almost diagonal, and that for all these matrices we count off-diagonal elements as of order ϵ. According to this counting rule, both the SM and the SUSY contributions to this process are of order ϵ^2, and here we have for the first time presented a complete result of SUSY effects at order ϵ^2.

Moreover, for reasons related to the necessary presence of $SU(2)_L$-breaking effects in the effective $Z\bar{s}d$ vertex, the supersymmetric contributions generated at the first order in the mass-insertion always appear suppressed by off-diagonal elements of the *chargino* mass matrix. These vanish as $\mathcal{O}(m_W/M_S)$ in the limit of a large supersymmetric scale, M_S, thus providing an additional damping factor. This suppression as well as the CKM one can only be avoided considering a double LR mixing in the up-squark sector.

We have performed a numerical analysis of the present bounds on the off-diagonal LR elements of the up-squark mass matrix relevant to the effective $Z\bar{s}d$ vertex. As a result, we have found that to our present knowledge the term which had been neglected so far (i.e. the one generated at second order in the mass-insertion approximation) is the most dangerous one, and could lead to very large enhancements in rare kaon decay rates. We have shown that the $K^+ \to \pi^+\nu\bar{\nu}$ rate could be enhanced up to one order of magnitude with respect to the SM prediction, whereas the neutral decay mode $K_L \to \pi^0\nu\bar{\nu}$ could be enhanced by up to two orders of magnitude. The same two orders of magnitude enhancement could be produced also in the decay $K_L \to \pi^0 e^+e^-$. Finally, we have also briefly discussed why the supersymmetric box contributions to these decays can be neglected as long as we are interested in potentially large effects.

Our results show that the current experimental efforts in the search for these rare decays are very much welcome and could give us valuable information on the flavor structure of the soft-breaking terms of a generic supersymmetric extension of the SM. Interestingly, we will not have to wait too long before experiments will reach the sensitivity necessary to observe, or at least to constrain, these supersymmetric effects. Indeed a preliminary evidence of the $K^+ \to \pi^+\nu\bar{\nu}$ decay has been recently obtained [16] and the BNL-E787 Collaboration is already analyzing new data on this mode. A sensitivity on $B(K_L \to \pi^0 e^+e^+)$ at the level of 10^{-10} is expected in few years by the KTeV experiment at Fermilab [22]. Finally, concerning the challenging $K_L \to \pi^0\nu\bar{\nu}$

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channel, while waiting for the dedicated experiments aiming to reach a sensitivity of 10^{-12} [23], even a non-dedicated experiment like KLOE [24] has a chance to give new and valuable information on possible extensions of the SM, since it can reach a sensitivity of 10^{-9} [25].

Acknowledgments

We thank S. Bellucci for participation in the early stage of this work. We are grateful also to N. Arkani-Hamed, F. Bossi and M. Worah for interesting discussions. G.I. acknowledges the hospitality of the Theory Group at SLAC, where part of this work has been done. This project is partially supported by the EEC-TMR Program, Contract N. CT98-0169.

A. Expansion of the mass matrices around the diagonal

Here we report the formulae needed to make the expansion around the diagonal of the mass matrices up to second order, i.e. including two mass insertions. Given an $n \times n$ Hermitian matrix M, we can decompose it in the form

$$M = M^0 + M^1 \,, \tag{A.1}$$

where $M^0 = \mathrm{diag}(m_1^0, \ldots, m_n^0)$ and M^1 has no elements on the diagonal. M can be diagonalized by a unitary matrix X, such that $XMX^\dagger = \mathrm{diag}(m_1, \ldots, m_n)$. Then, if f is an arbitrary function, we have

$$\begin{aligned} X^\dagger_{ik} f(m_k) X_{kj} = {} & \delta_{ij} f(m_i^0) + M^1_{ij} f(m_i^0, m_j^0) + \\ & + M^1_{ik} M^1_{kj} f(m_i^0, m_j^0, m_k^0) + O\left((M^1)^3\right) , \end{aligned} \tag{A.2}$$

where we have adopted the notation of Buras, Romanino and Silvestrini [8] to define an n-argument function from an $n-1$-argument one:

$$f(x, y, z_1, \ldots, z_{n-2}) = \frac{f(x, z_1, \ldots, z_{n-2}) - f(y, z_1, \ldots, z_{n-2})}{x - y} \,. \tag{A.3}$$

B. Loop functions

The loop functions appearing in the top-quark penguin diagrams discussed in section 2 are given by

$$C(x) = \frac{x}{8} \left(\frac{x-6}{x-1} + \frac{3x+2}{(x-1)^2} \log x \right) , \tag{B.1}$$

$$H(x) = \frac{x^2}{8} \left(-\frac{\log x}{(x-1)^2} + \frac{1}{x-1} \right) . \tag{B.2}$$

The multi-variables functions $k(x_1,\ldots,x_n)$, $j(x_1,\ldots,x_n)$ and $l(x_1,\ldots,x_n)$, occurring in chargino-squark diagrams, are defined according to the recursive formula given in (A.3). The explicit expression of the single-variable functions are

$$j(x) = \frac{x\log x}{x-1}\,, \qquad k(x) \;=\; x\,j(x)\,, \tag{B.3}$$

$$l(x) = k\left(\frac{1}{x},\,\frac{1}{x}\right) - \frac{2}{x}j\left(\frac{1}{x},\,\frac{1}{x}\right) - k\left(\frac{1}{x},\,\frac{x_{u_L1}}{x}\right) - k\left(\frac{1}{x_{u_L1}},\,\frac{x}{x_{u_L1}}\right). \tag{B.4}$$

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